

SEMINAR (SS15, FR 14-16, M006)
HOMOTOPICAL ALGEBRA - MODEL CATEGORIES

G. RAPTIS

Schedule

- **Basic theory of model categories**

- **Talk 1 (17.04.2015): The definition of a model category and basic properties.** The axioms of a model category. The retract argument and its consequences. Ken Brown’s lemma (with proof). Examples of model categories (without proof). Generalities about localizations of categories with weak equivalences. The main references are [Ho, 1.1.1-1.2.3] and [DS, §3]. See also [Re] for another example which is worth mentioning.
- **Talk 2 (24.04.2015): Homotopies and the homotopy category.** Cylinder objects and path objects. Left and right homotopies and their properties. The “Whitehead” theorem. The homotopy category of a model category. The main references are [Ho, 1.2.4-1.2.10], [Hi, 7.3-7.5, 8.3], [Qu, §1].
- **Talk 3 (08.05.2015): Quillen adjunctions and derived functors.** Left and right Quillen functors and their properties. Total left and right derived functors and their universal properties. Quillen equivalences. Equivalent characterizations of Quillen equivalences. The main references are [Ho, 1.3], [Hi, 8.4-8.5]. See also [Qu, §4].

- **A method for recognizing model categories**

- **Talk 4 (15.05.2015): The small-object argument.** Notions of smallness of objects. Injectives, cell complexes, cofibrations and fibrations with respect to a class of morphisms. The small-object argument. The main references are [Ho, 2.1.1-2.1.16] and [Hi, 10.1-10.5].
- **Talk 5 (22.05.2015): Cofibrantly generated model categories.** The definition of a cofibrantly generated model category and basic properties. The recognition theorem for cofibrantly generated model categories. Application to categories of diagrams in a cofibrantly generated model category and to the existence of homotopy colimits. The main references are [Ho, 2.1.17-2.1.21] and [Hi, 11.1-11.3, 11.5-11.6].

- **Examples**

- **Talk 6 (29.05.2015): Chain complexes.** The proof of the (projective) model structure on the category of chain complexes of R -modules. Examples of left/right Quillen functors and their derived functors. Other model category structures on the

category of chain complexes and further generalizations (without proofs). The main reference is [Ho, 2.3]. See also [DS, §7] and [GS].

- **Talk 7 (05.06.2015): Topological spaces.** The proof of the standard model structure on the category of topological spaces. Examples of left/right Quillen functors and their derived functors. The Strøm model structure on topological spaces (to be mentioned without proof). Other related model categories (without proofs). The main reference is [Ho, 2.4]. See also [DS, 3.6, §8].

- **Simplicial homotopy theory**

- **Talk 8 (12.06.2015): Simplicial sets.** Basic definitions and properties. (Trivial) (co)fibrations of simplicial sets. The geometric realization functor. Weak equivalences of simplicial sets. The main references are [Ho, 3.1-3.2] and [GJ, I.1-I.4].
- **Talk 9 (19.06.2015): Mapping spaces and (combinatorial) homotopy groups.** Mapping spaces and (co)fibrations. Homotopy groups of a fibrant simplicial set. Long exact sequence of homotopy groups. The main references are [Ho, 3.3-3.4] and [GJ, I.5-I.7].
- **Talk 10 (26.06.2015): The model category of simplicial sets.** Fibrations and geometric realization. Outline of the proof of the model structure on the category of simplicial sets. The Quillen equivalence between simplicial sets and topological spaces. The main references are [Ho, 3.5-3.6] and [GJ, I.10-I.11].

- **Special types of model categories and additional structure**

- **Talk 11 (03.07.2015): Simplicial model categories.** The axioms of a simplicial model category. Examples and non-examples. Mapping spaces and weak equivalences. Simplicial homotopy and left/right homotopy. Simplicial model categories of simplicial objects in a category. Homology and cohomology. The main references are [GJ, II.2-II.5], [Hi, Chapter 9] and [Qu, II.4-II.5]. See also [GS].
- **Talk 12 (10.07.2015): Pointed model categories.** The suspension and loop functors. Cofiber and fiber sequences and their properties. Stable model categories. The main references are [Ho, 6.1-6.3, 6.5, 7.1] and [Qu, I.2-I.3].
- **Talk 13 (17.07.2015): (tba)**

REFERENCES

- [DS] W. G. DWYER; J. SPALIŃSKI, *J. Homotopy theories and model categories*. Handbook of algebraic topology, pp. 73-126, North-Holland, Amsterdam, 1995.
- [GJ] P. G. GOERSS; J. F. JARDINE, *Simplicial homotopy theory*. Progress in Mathematics, 174. Birkhäuser Verlag, Basel, 1999.
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- [Hi] P. S. HIRSCHHORN, *Model categories and their localizations*, Mathematical Surveys and Monographs, Vol. 99, Amer. Math. Soc., 2003.
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- [Qu] D. QUILLEN, *Homotopical Algebra*, Lect. Notes in Math., Vol. 43, Springer, 1967.
- [Re] C. REZK, *A model category for categories*, www.math.uiuc.edu/~rezk/cat-ho.dvi

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