

Seminar:
Catastrophe Theory – Katastrophentheorie
(Singularitäten differenzierbarer Abbildungen)
(WS23/24 20, Mi 12-14, M103)

SEMINAR SCHEDULE

- **Talk 1 (18.10.2023): Introduction and Recollections.** Introduction to the study of germs of smooth functions and the notions of stability and genericity (via examples). Recollections on regular values, Sard's theorem (without proof) and the local description of a submersion (or any germ of constant rank). Classification of critical points of functions of one variable. The material can be found in [CH, Chapter 1], [Br, Chapters 1–2], and [Lu, 1.1–1.4].
- **Talk 2 (25.10.2023): The Morse Lemma.** The definition of a non-degenerate critical point. The local description of smooth functions near a non-degenerate critical point. Discussion of the stability and the genericity of Morse functions. The material can be found in [CH, Chapter 1] and [Lu, 1.5 and 2.4]. See also [PS, Chapter 4].
- **Talk 3 (08.11.2023): The Reduction (or Splitting) Lemma.** The definition of the corank of a smooth function at a point. The Reduction (or Splitting) Lemma (with proof). Examples. The main reference is [CH, Chapter 3]. See also [PS, Chapter 4] and [Br].
- **Talk 4 (15.11.2023): Germs and Jets.** The algebra of germs of smooth functions $\mathbb{R}^n \rightarrow \mathbb{R}$ at the origin. The lemma of Borel. Equivalence and k -equivalence of germs. Determinacy and k -determinacy of a germ and invariance of these properties under equivalences. Examples. The main references are [CH, pp. 65–75], [Br, Chapters 3–4] and [Lu, 3.2].
- **Talk 5 (22.11.2023): Determinacy of Germs I.** The Jacobi ideal of a germ. The Nakayama Lemma. Mather's sufficient criterion for determinacy (with sketch of proof) and its consequences. Examples. The main reference are [CH, pp. 75–84], [Br, Chapter 11] and [Lu, 3.3]. See also [PS, Chapter 8].
- **Talk 6 (29.11.2023): Determinacy of Germs II.** The Linearization Lemma (without proof). Mather's necessary condition for determinacy (with proof) and its consequences. Examples. The main reference are [CH, pp. 85–100], [Br, Chapter 11] and [Lu, 3.3]. See also [PS, Chapter 8].
- **Talk 7 (06.12.2023): Codimension.** The codimension of a germ. Relations between the codimension of a germ, its determinacy and its corank. Examples. The main reference

is [CH, Chapter 5]. See also [Br, Chapter 13] and [Lu, 3.3–3.4].

- **Talk 8 (13.12.2023): Summary and Examples.** A summary of the results so far (with examples). See [CH, Chapters 1–5], [Lu, 1.1–2.4, 3.1–3.3, 4.1–4.4], [PS].
- **Talk 9 (20.12.2023): Classification of Germs.** Thom’s theorem on the classification of degenerate critical points of codimension at most 4. The main reference is [CH, Chapter 6]. See also [Lu, Appendix II] and [Br, Chapter 15].
- **Talk 10 (10.01.2024): Unfoldings.** The definition of an unfolding of a germ. The catastrophe and bifurcation sets. Examples. Induced unfoldings. Equivalences of unfoldings. Sums of unfoldings. The definition of a (uni)versal unfolding. The main reference is [CH, Chapter 7]. See also [Br, Chapter 14], [Lu, pp. 66–77] and [PS, Chapter 7].
- **Talk 11 (17.01.2024): Transversality.** The definition of a k -transversal unfolding. Existence of k -transversal unfoldings and comparison with versal unfoldings. Statement of the Malgrange-Mather Preparation Theorem (without proof) and discussion of some of its consequences. Uniqueness of k -transversal unfoldings of a k -determined germ (*‘Main Lemma’*) (without proof). The main references are [CH, pp. 169–172, 177–178, 197–202] and [Br, Chapter 16]. See also [Lu, 3.4].
- **Talk 12 (24.01.2024): Universal Unfoldings.** Thom’s fundamental theorem on the existence and uniqueness of universal unfoldings of a finitely determined germ. The main references are [CH, Chapter 10], [Br, Chapter 16] and [Lu, 3.4]. See also [PS, Chapters 7 and 8] for an informal introduction.
- **Talk 13 (31.01.2024): Examples of Elementary Catastrophes.** Discussion of the universal unfoldings of some of the 7 elementary catastrophes. The main references are [Br, Chapter 17], [Lu, Chapter 4] and [PS, Chapters 7 and 9].
- **Talk 14 (07.02.2024): *tbd***

REFERENCES

- [Br] TH. BRÖCKER, *Differentiable germs and catastrophes*. Translated from the German, last chapter and bibliography by L. Lander. London Mathematical Society Lecture Note Series No. 17. Cambridge University Press, Cambridge-New York-Melbourne, 1975.
- [CH] DOMENICO P. L. CASTRIGIANO; SANDRA A. HAYES, *Catastrophe theory*. Second edition. With a foreword by René Thom. Westview Press, Advanced Book Program, Boulder, CO, 2004.
- [Lu] YUNG-CHEN LU, *Singularity theory and an introduction to catastrophe theory*. Universitext, Springer-Verlag, New York-Berlin, 1980.
- [PS] TIM POSTON; IAN STEWART, *Catastrophe theory and its applications*. With an appendix by D. R. Olsen, S. R. Carter, and A. Rockwood. Surveys and Reference Works in Mathematics No. 2. Pitman, London-San Francisco, Calif.-Melbourne, 1978.