WORKSHOP: ALGEBRAIC K-THEORY OF SPACES

UNIVERSITY OF REGENSBURG, 24–28 JULY 2023

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
9.00 – 10.00	Registration		Klein		
10.00 – 11.00	Arone	Arone	Steimle	Arone	Klein
11.00 – 11.30	break	break	break	break	break
11.30 – 12.30	Malkiewich & Merling	Malkiewich & Merling	Flores, Kirstein	Malkiewich & Merling	Steimle
12.30 – 14.15	lunch	lunch	lunch	lunch	
14.15 – 15.15	Varisco	Varisco	free afternoon	Varisco	
15.15 – 15.45	break	break		break	
15.45 – 16.45	Krannich	Krannich		Krannich	
16.45 – 17.00	break	break		break	
17.00 – 18.00	Lehner, Semikina	Muñoz-Echániz, Yeakel		Davis, Kallel	

All talks will take place in Lecture Hall H31. The registration and coffee breaks will be in Seminar Room M101.

Invited lecture series

Gregory Arone: The rank filtration of topological and algebraic K-theory.

- *Lecture 1*: The stable rank filtration of algebraic K-theory was defined by J. Rognes in 1992. Rognes's definition was based on Waldhausen's S.-construction. Later, Arone and Lesh studied a variant of the filtration, based on Segal's Gamma-space construction. The two filtrations are equivalent for topological K-theory, but not for algebraic K-theory. We will introduce the two constructions in parallel and show how to compare them. Rognes made a well-known conjecture about the connectivity of the subquotients in the rank filtration. We will show how our construction can be used to prove a version of Rognes's conjecture for topological K-theory. If time permits, we will say something about recent progress on the conjecture by Galatius–Kupers–Randal-Williams and by Miller–Patz–Wilson.
- *Lecture 2*: I will tell about a joint work with Ilan Barnea and Tomer Schlank. We define a stable homotopy category of C^*-algebras, analogous to the classical stable homotopy category of spaces. By the Schwede–Shipley theorem, it is equivalent to the category of spectral presheaves on the full subcategory whose objects are matrix algebras. We define a natural "rank" filtration of the category of matrix algebras and stable maps between them. This filtration is a natural lift of the rank filtration of complex K-theory. We

describe the subquotients of this filtration in terms of complexes of direct-sum decompositions. As an application, we describe the rationalization of (our version of) the stable homotopy category of C^*-algebras, and obtain some information about its p-localization. There are also some intriguing consequences regarding its chromatic localization.

- *Lecture 3*: In this lecture we will show how the rank filtration gives rise to a minimal (or "near minimal") projective resolution of the connective complex K-theory spectrum. This answers a 40-year-old question of Nick Kuhn. The result is analogous to a theorem of Kuhn that the symmetric powers filtration gives rise to a minimal projective resolution of the Eilenberg–Mac Lane spectrum. There are some connections to functor calculus and the theorems of Kuhn and Mark Behrens on the Taylor tower of the identity evaluated at S^1.

John Klein.

- *Lecture 1*: **Bundle lifting problems and the algebraic K-theory of spaces**
  Given a fibration with homotopy finite fibers, does there exist a fiber bundle with compact manifold fibers having the same fiber homotopy type? This talk is based on some of my earlier work with Bruce Williams.
- *Lecture 2*: **Poincaré complex diagonals and the Bass trace conjecture**
  In this talk I will explain my recent work with Florian Naef on the problem of when a Poincaré duality space admits a "tubular neighborhood" inside its diagonal.

Manuel Krannich: Diffeomorphism groups of discs. (3 lectures)
In the late 70s, based on work of Borel on the cohomology of arithmetic groups and of Waldhausen on the relation between manifolds and algebraic K-theory, Farrell and Hsiang computed the rational homotopy groups of the space of diffeomorphisms of a closed d-dimensional disc in a range of degrees up roughly $(1/3)\cdot d$. They discovered that in this range these groups either vanish or agree with a shift of the rational K-groups of the integers, depending on the parity of d. In joint work with Oscar Randal-Williams, we computed these groups in a range up to roughly $(3/2)\cdot d$. Two further phenomena in addition to the contribution from algebraic K-theory occur in this range, one related to Weiss' "surreal" Pontryagin classes and one to Kontsevich's configuration space integrals as studied by Watanabe. In this series of lectures, I will motivate why one is interested in knowing these rational homotopy groups in the first place, explain the three sources of classes, and give an outline of how our computation goes.

Cary Malkiewich and Mona Merling: Equivariant A-theory and equivariant h-cobordisms.

- *Lecture 1*: **Equivariant K-theory and A-theory**
  Algebraic K-theory is an invariant of ring spectra, and more generally Waldhausen categories, that takes values in spectra. If the input is a ring spectrum with G-action, or more generally a Waldhausen category with G-action, we show how to produce a genuinely equivariant spectrum as a result. The key constructions are a categorical version of homotopy fixed points, and the technology of spectral Mackey functors. The most natural construction from a category with G-action, only gives a "coarse" version of equivariant A-theory and we need to work a little harder to promote this

to a construction that has a tom Dieck-style splitting on the fixed point spectra.

- *Lecture 2*: **Equivariant h-cobordisms**
  Parallel to equivariant A-theory is the theory of equivariant cobordisms on a G-manifold M. This theory becomes rather subtle when you try to explain how it is a functor along equivariant manifolds and equivariant smooth embeddings. In this talk, we show how to address this functoriality in a satisfactory way, producing a geometric model for the equivariant Whitehead space. As a consequence, we get a splitting on the stable space of equivariant h-cobordisms, parallel to the tom Dieck-style splitting on equivariant A-theory.
- *Lecture 3*: **The equivariant assembly map**
  The fundamental theorem concerning algebraic K-theory of spaces is the parametrized h-cobordism theorem. It says that stable homotopy includes into A-theory as a summand, and the homotopy fiber of this inclusion is the stable space of smooth h-cobordisms. In this talk we show how to prove the equivariant version of this theorem, using the tom Dieck-style splittings that exist on both A-theory and the stable h-cobordism space.

Wolfgang Steimle: The visible symmetric signature. (2 lectures)
The visible symmetric signature is a powerful invariant of closed manifolds and Poincaré duality spaces, defined by Weiss–Williams. It combines Euler characteristic and signature in their most general forms (taking values in algebraic K-theory and L-theory repectively) in one invariant, and plays a key role in the classification of automorphisms of manifolds.

Recent works on the foundations of hermitian K-theory allows for a considerable clarification of the visible symmetric signature: It can be seen as an invariant taking values in hermitian K-theory of parametrized spectra. Our goal in this lecture series is to explain this modern definition of the visible symmetric signature and its role in the classification program of Weiss–Williams.

Everything is based on joint works with Calmès, Dotto, Harpaz, Hebestreit, Land, Moi, Nardin and Nikolaus.

Marco Varisco: Assembly Maps and Isomorphism Conjectures. (3 lectures)
Assembly maps are fundamental tools for studying the algebraic K-theory of group rings, and more generally group ring spectra. Motivated by their work on rigidity of manifolds, Farrell and Jones conjectured that certain assembly maps are isomorphisms, and proved it in many important cases. I will introduce assembly maps and the Farrell–Jones conjecture, and survey the current status and some of the far-reaching consequences in algebra and manifold topology of this conjecture. I will then talk about the injectivity half of the conjecture, and about the relation between spherical and integral coefficients.

Contributed talks

Jim Davis: A remark on the fundamental theorem of algebraic K-theory.
If F is a functor from Rings to Abelian groups, then Bass defined the functor NF from Rings to Abelian groups as the kernel$(F(R[x]) \to F(R))$, induced by the ring homomorphism given by evaluation at 1. The Nil groups $NK_q(R)$ appear in Bass/Quillen's fundamental theorem of algebraic K-theory. Since they vanish for regular rings, they measure regularity of the ring. Cortiñas, Haesemeyer, and Weibel proved that if R is a commutative $\mathbb{Q}$-algebra, that the iterated Nil group N^nK_q is an infinite direct sum of groups $NK_{q-i}(R)$ where $n-1 \geq i \geq 0$.

I later conjectured that this is true for any ring. A corollary is the computation of the K-theory of a polynomial ring $K_q(R[x_1, \ldots, x_n])$ as $K_q(R) \oplus_i (NK_{q-i}(R))^\infty$.

I recently proved this conjecture, using results of Davis–Quinn–Reich, the Farrell–Jones Conjecture, and a trick.

Ramon Flores: Bredon homology and K-theory for Artin groups of dihedral type.
For Artin groups of dihedral type, we will show how to compute the Bredon homology groups of the classifying space for the family of virtually cyclic subgroups with coefficients in the K-theory of a group ring, and describe the relations of these computations with algebraic K-theory in the context of the Farrell–Jones conjecture.

Sadok Kallel: Combinatorial invariants of stratified spaces.
We extend the well-known construction of the Grothendieck ring of varieties to categories whose objects can be partitioned into predefined strata (i.e. stratifiable spaces). To this end, we introduce "Lego categories" which are subcategories of the category of spaces stratifiable by locally compact strata in Euclidean space. Restricting to strata that are cohomologically of finite type, as well as their closures, we construct a well-defined motivic morphism on the associated Grothendieck ring which coincides with the Euler characteristic with compact supports on locally compact spaces.

This "categorification" allows streamlined combinatorial derivations of Euler characteristics, be they topological or with compact supports. Main applications pertain to spaces stratified by configuration spaces, with new results including the computation of the Grothendieck class of graph configuration spaces, of orbit configuration spaces and of finite subset spaces.

Dominik Kirstein: A twisted Bass–Heller–Swan decomposition for localising invariants.
Classically, the Bass–Heller–Swan decomposition relates the algebraic K-theory of a Laurent polynomial ring to the algebraic K-theory of its coefficient ring. In this talk, I will explain how to obtain a generalisation of this result in the setting of localising invariants of stable infinity-categories that come with an automorphism. As an application, one gets a splitting result for Waldhausen's A-theory of mapping tori. This is joint work with Christian Kremer.

Georg Lehner: The passage from the integral to the rational group ring in algebraic K-theory.
Let G be a group, $\mathbb{Z}G$ and $\mathbb{Q}G$ be the integral and rational group ring. A known theorem due to Swan states that if G is a finite group and P a finitely generated projective $\mathbb{Z}G$-module, the rationalization $P \otimes \mathbb{Q}$ is free as a $\mathbb{Q}G$-module. There have been many attempts to generalize Swan's theorem to infinite groups, notably the Bass

trace conjectures. Lück–Reich formulated what is known as the integral $K_0(\mathbb{Z}G)$-to-$K_0(\mathbb{Q}G)$-conjecture, which states that the map induced by rationalization from $K_0(\mathbb{Z}G)$ to $K_0(\mathbb{Q}G)$ has image only in the subgroup of $K_0(\mathbb{Q}G)$ generated by the free modules. We show that the integral $K_0(\mathbb{Z}G)$-to-$K_0(\mathbb{Q}G)$-conjecture is false by constructing concrete counterexamples via amalgamated products of groups K and H, which allow particular quaternionic representations.

Samuel Muñoz-Echániz: A Weiss–Williams result for spaces of embeddings and the homotopy type of spaces of long knots.
A celebrated theorem of Weiss and Williams expresses (in a range of homotopical degrees) the difference between the spaces of diffeomorphisms and block diffeomorphisms of a manifold in terms of its algebraic K-theory. In this talk, I will present an analogous result for embedding spaces. Namely, for M a manifold of dimension at least 5 and P submanifold of M of codimension at least 3, we describe the difference between the spaces of block and ordinary embeddings of P into M as a certain infinite loop space involving the relative algebraic K-theory of the pair $(M, M - P)$. The range of degrees in which this description applies is the so-called concordance embedding stable range which, by recent developments of Goodwillie–Krannich–Kupers, is far beyond that of the aforementioned theorem of Weiss–Williams.

I will also explain how one can use this result to give a full description of the homotopy type (away from 2 and roughly up to the concordance embedding stable range) of the space of long knots of codimension at least 3, that is, embeddings rel boundary of the p-disk into the d-disk for $d - p > 2$ and $d > 4$.

Julia Semikina: K-theory of manifolds and cobordisms.
The generalized Hilbert's third problem asks about the invariants preserved under the scissors congruence operation: given a polytope P in $\mathbb{R}^n$, one can cut P into a finite number of smaller polytopes and reassemble these to form Q. Kreck, Neumann and Ossa introduced and studied an analogous notion of cut and paste relation for manifolds called the SK-equivalence ("schneiden und kleben" is German for "cut and paste"). In this talk I will explain the construction that will allow us to speak about the "K-theory of manifolds" spectrum. The zeroth homotopy group of the constructed spectrum recovers the classical groups SK_n. I will show how to relate the spectrum to the algebraic K-theory of integers, and how this leads to the Euler characteristic and the Kervaire semicharacteristic when restricted to the lower homotopy groups. Further I will describe the connection of our spectrum with the cobordism category.

Sarah Yeakel: Isovariant homotopy theory.
Equivariant maps which preserve isotropy groups are called isovariant, and they show up in equivariant surgery theory and other settings when homotopy theory is applied to geometry. For a finite group G, we will discuss a homotopy theory on the category of G-spaces with isovariant maps, as well as an application to fixed point theory. This is joint work with Inbar Klang.